

102 學年度交通大學應用數學系博士班入學考離散數學試題

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There are 7 problems listed below. You have to explain your answers in more detail in order to obtain full credits. (15 points each)

1. Show that if G is a connected bipartite graph which has 6 edges, then G contains an induced subgraph which has exactly 3 edges.
2. Prove that if G is a graph with n vertices which has no 3-cycles, then G has at most $\lfloor n^2/4 \rfloor$ edges. ($\lfloor \cdot \rfloor$ stands for Gaussian notation.)
3. Let $S = \{1, 2, \dots, 13\}$. Find a collection \mathcal{B} of 13 4-subsets such that every 2-subset of S is contained in exactly one subset in \mathcal{B} .
4. Find the number of distinct DNA sequences of length 8 such that AT and TA do not occur in the sequence. (A DNA sequence is a sequence whose elements are A, C, G and T.)
5. Let A be a set of 13 distinct real numbers. Prove that there exist two elements x and y in A such that $0 < \frac{x-y}{1+xy} \leq 2 - \sqrt{3}$.
6. Let $\{a_n\}$ be a sequence of positive integers such that $a_0 = a_1 = 2$, and $a_{n+1} = a_n a_{n-1}$, $n \geq 2$. Find a_{100} .
7. For each positive integer $n \geq 3$, prove that $\varphi(n)$ is an even integer where $\varphi(n)$ is the number of positive integers which are smaller than n and relatively prime with n .