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National Chiao Tung University

Department of Applied Mathematics

Discrete Mathematics Qualifying Examination

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Problem 1.(15pts) Let $\{A_1, A_2, \ldots, A_m\}$ be a collection of m distinct subsets of $\{1, 2, \ldots, n\}$, where $|A_i| \leq n/2$ for $i = 1, \ldots, m$, with the property that any two of the subsets have a nonempty intersection. Prove that

$$\sum_{i=1}^{m} \frac{1}{\binom{n-1}{|A_i|-1}} \leq 1.$$

Problem 2.(15pts) Let A be a $(0,1)$-matrix with entries a_{ij}. By a *line*, we mean a row or column of A. Prove that the minimum number of lines of A that contain all the 1's of A is equal to the maximum number of 1's in A, no two on a line.

Problem 3.(15pts) Let m be given. Prove that if n is large enough, every $n \times n$ $(0,1)$-matrix has a principal submatrix of size m, in which all the elements below the diagonal are the same, and all the elements above the diagonal are the same.

Problem 4.(15pts) Let $a_1, a_2, \ldots, a_{n^2+1}$ be a permutation of the integers $1, 2, \ldots, n^2 + 1$. Prove that Dilworth's theorem implies that the sequence $a_1, a_2, \ldots, a_{n^2+1}$ has a subsequence of length $n+1$ that is monotone.

Problem 5.(15pts) Find the number of sequences $a_1, a_2, \ldots, a_{2n}$ of $2n$ terms that can be formed by using exactly n 1's and exactly n -1's such that $a_1 + a_2 + \cdots + a_k \geq 0$, $(k = 1, 2, \ldots, 2n)$. Show your work.

Problem 6.(15pts) Determine the number of n-digit numbers with each digit odd, where the digits 1 and 3 occur an even number of times. Prove your answer.

Problem 7.(10pts) Prove that if an $S(3, 6, v)$ exists, then $v \equiv 2$ or $6 \pmod{20}$.