博士班資格考(設計理論)

1. (1) State the Fishers' inequality and the Bruck-Ryser-Chowla theorem. (5% each)
   (2) Prove the Fishers' inequality. (10%)

2. Give an explicit construction of a projective plane of order 5 and its related design. (20%)

3. Construct explicitly an Hadamard matrix of order 20. (20%)

4. Prove that: If q is a prime power, then there exist $q-1$ MOLS (mutually orthogonal Latin squares) of order q. (20%)

Definition. A *group divisible design* GDD(v, k, m) consists of a collection of m-subsets, called *groups*, of a v-set S and a collection of k-subsets, called *blocks*, such
(i) the groups form a partition of S,
(ii) each pair of elements from different groups occur together in exactly one block,
(iii) no block contains two elements from the same group.

5. Let $p(x)=x^4+x+1$. Then $p(x)$ is primitive over GF(2). If α is a root of $p(x)$ in an extension field of GF(2), then the set $\{0,1,\alpha,\alpha^2,\ldots,\alpha^{14}\}$ forms a finite field of order 16. Let $S=\{1,2,\ldots,14\}$, $G=\{\{a,b\}\mid\alpha^a+\alpha^b=1\}$, and $B=\{\{a,b,c\}\mid\alpha^a+\alpha^b+\alpha^c=1\}$. Show that ($S$, G, B) forms a group divisible design GDD(14, 3, 2). (20%)