博士班資格考－設計理論 97.2月

Work out all problems. Detail arguments should be provided.

[12%] 1. Let H be a normalized Hadamard matrix of order m with $m \geq 2$. Show that each row (and each column), other than the first, has exactly half its entries equal to $+1$.

[20%] 2. (a) Define the following terms: nets, transversal designs, orthogonal Latin squares, and strongly regular graphs.

(b) Describe the relationship among the above combinatorial structures, e.g. how to obtain one from another.

[20%] 3. (a) Does there exist a $(29, 8, 2)$ difference set? If yes, find one; otherwise, explain why.

(b) Find at least 3 difference sets or difference systems using the cyclic group of order 29. Can you find more?

[24%] 4. Let $S = \{1, 2, \ldots, v\}$ and let T be a set of 3-element subsets of S. Suppose that each pair of distinct elements of S belongs to at least one triple in T, and $|T| \leq v(v-1)/6$.

(a) Show that (S, T) is a Steiner triple system.

(b) Define $A = (a_{ij})$ by $a_{ii} = i$ and, if $i \neq j$, $a_{ij} = k$ where $\{i, j, k\}$ is the unique block containing both i and j. Show that A is a Latin square with $a_{ij} = a_{ji}$ for all i and j.

[24%] 5. Let $S_1, S_2, \ldots, S_t$ be a set of mutually orthogonal Latin squares of order $n \geq 3$.

(a) Show that $t \leq n - 1$.

(b) If each S_i, $1 \leq i \leq t$, is idempotent (i.e. the diagonal of S_i is $[1, 2, \ldots, n]$), then $t \leq n - 2$.