

NCTU Department of Applied Mathematics

Qualifying Examination in Discrete Mathematics

for the Ph. D. Program

September 2018

Note: The proofs and statements must be detailed. When you quote some theorems, please prove them.

1. Let U be the set of all permutations (a,b,c,d,e,f) on $\{1,2,3,4,5,6\}$ with $a \notin \{1,2\}$, $b \notin \{2,3\}$, $c \notin \{3,4\}$, $d \notin \{4,5\}$, $e \notin \{5,6\}$, and $f \notin \{6\}$. Find the cardinality of U . (20%)
2. Prove the value of a maximum flow is equal to the value of a minimum cut in a transportation network. (20%)
3. Show that there exist four mutually orthogonal Latin squares of order 5 if and only if there exists a 2- $(25,5,1)$ -design. (20%)
4. Prove that if G is a planar graph, then the chromatic number of G is less than or equal to 5. (20%)
5. Let P be a partially ordered set. Prove that if P can be partitioned into m disjoint chains and P has an antichain with n elements, then $m \geq n$. (10%)
6. Let n be a positive integer. Prove that the binomial identity

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n} \text{ by a generating function.} \quad (10\%)$$